part to the generation of the high lift coefficients, which some insects require to sustain flight.

References

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Response of a Symmetric Missile in a Spin-Varying Environment

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Nomenclature

$A_{1,2}$ = constants
a_n = coefficients in a power series expansion
b = $(M_q\tau_\infty/I + i\omega_0)/c$
c = L_p/I_x
C_D = drag coefficient
C_{L_α} = lift coefficient
C_{M_α} = static moment coefficient
$C_{M_{p\alpha}}$ = Magnus moment coefficient
C_{l_p} = roll damping moment coefficient
$C_{M_{\dot\alpha}}, C_{M_q}$ = damping moment coefficients
I_x, I = axial, transversal moments of inertia
i = $\sqrt{-1}$
K_j = amplitude of j mode ($j = 1,2$)
L_p = $\rho V S \ell^2 C_{l_p}/4$
m = mass
M_α = $\rho V^2 S\ell C_{M_\alpha}/2$
M_q = $\rho V S\ell^2 C_{M_q}/4$
n = integer
P = $d\phi/dt$, roll rate
P_l = $P_\infty - P$
q = complex angular velocity $(q = i\tilde r + \tilde q)$
$\tilde q, \tilde r$ = angular velocity components
R = iI_x/Ic
r_j = $[M_q(1 \pm \tau_\infty)/2I + i(\eta \pm \omega_0)]/c$
S = reference area
t = time
V = magnitude of velocity
$\alpha_l \beta_l$ = angles of attack and sideslip
α = $\beta_l + i\alpha_l$
η = $I_x P/2I$
λ_j = damping rate of the j mode amplitude $\dot K_j/K_j$
ρ = air density
ϕ = roll angle
ϕ_j = j-modal phase angle
Φ_j = degenerate hypergeometric functions
τ = η/ω
ω = $(\omega_0^2 + \eta^2)^{1/2}$
ω_0 = $(M_\alpha/I)^{1/2}$

Superscripts

$(\ )'$ = primes denote derivatives with respect to s
$(\dot{\ })$ = dots denote derivatives with respect to t
$(\ )^*$ = stars denote derivatives with respect to P_l

Subscripts

0 = initial value
∞ = value at $t = \infty$
n = index
e = envelop

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Approximate Solution

THE response characteristics of a fin-stabilized symmetric missile for slowly time-varying parameters were formulated by C.H. Murphy.[1] Consider a motion about the center of gravity in which all flight parameters and aerodynamic coefficients are constant, but where the roll rate is varying according to

$$P = P_\infty + (P_0 - P_\infty)e^{ct} \tag{1}$$

where $c < 0$. Expressing Murphy's formulations in the time domain

$$\dot\phi_j = \eta \pm \omega \tag{2}$$

$$\dot\eta = (L_P/I_x)(\eta - \eta_\infty) \tag{3}$$

$$\ddot\phi_j = \dot\eta(1 \pm \tau) \tag{4}$$

and the damping rates are

$$\lambda_j = \frac{M_q}{2I}(1 \pm \tau) \mp \frac{L_p}{2I_x}(\tau - \tau_\infty)(1 \pm \tau) \pm \frac{M_{p\alpha}}{I_x}\tau \tag{5}$$

Equation (5) suggests that there might be combinations for which λ_j will become positive for some time interval. Consider a special case, in the absence of Magnus moment.

Decreasing roll rate flight for which $P_\infty = 0$, Eq. (5) becomes

$$\lambda_j = \frac{1 \pm \tau}{2I}\left(M_q \mp \frac{I}{I_x} L_p \tau\right) \tag{6}$$

For some P_0, λ_l might become positive, for some time interval. Equation (5) should also be considered when test data is analyzed to determine Magnus moment coefficient.

Exact Solution

The finding that λ_j might become positive due to roll damping terms was considered important, and an attempt to find an exact solution was made. Consider the increasing roll rate case ($P_0 = 0$), with the previous assumptions.

The equation of motion in aeroballistic axes is[2]

$$\ddot\alpha - \left(\frac{M_q}{I} + i\frac{I_x}{I}p\right)\dot\alpha + \omega_0^2\alpha = 0 \tag{7}$$

Change the independent variable from t to P_l

$$P_l = P_\infty e^{ct} \tag{8}$$

and get

$$P_l^2\alpha^{**} + \left[\left(1 - \frac{M_q}{Ic} - i\frac{I_x}{Ic}P_\infty\right)P_l + i\frac{I_x}{Ic}P_l^2\right]\alpha^* + \left(\frac{\omega_0}{c}\right)^2\alpha = 0 \tag{9}$$

α is assumed to be presented in power series of P_l

$$\alpha = P_l^r \sum_{n=0}^{\infty} a_n P_l^n \tag{10}$$

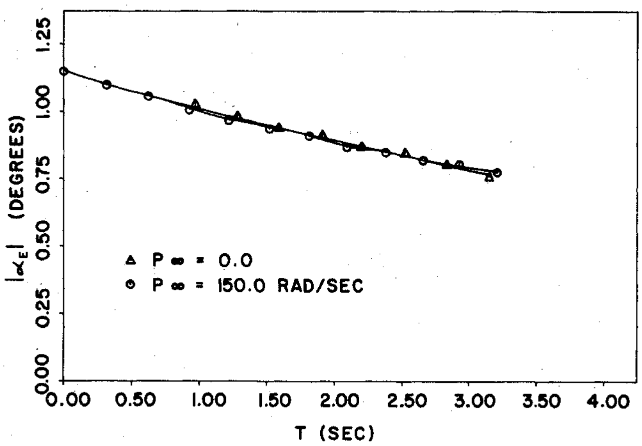

Fig. 1 The effect of P_∞ on $|\alpha_E|$.

Substitute Eq. (10) into Eq. (9) and evaluate r and the a_n's

$$r_j = [M_q(1 \pm \tau_\infty)/2I + i(\eta_\infty \pm \omega_0)]/c; \quad j=1,2 \tag{11}$$

and

$$\alpha(t) = A_1\Phi_1(r_1, 1+b; RP_1)e^{cr_1t} + A_2\Phi_2(r_2, 1-b; RP_1)e^{cr_2t} \tag{12}$$

where

$$R = i(I_x/I)(1/c) \tag{13}$$

$$b = (1/c)[(M_q/I)\tau_\infty + i\omega_0] \tag{14}$$

and,[3]

$$\Phi_{1,2}(r_{1,2}, 1 \pm b; RP_1 = 1 + \sum_{n=1}^{\infty} \frac{(RP_1)^n \prod_{k=1}^{n}(r_{1,2}+k-1)}{n! \prod_{k=1}^{n}(k \pm b)} \tag{15}$$

$A_{1,2}$ are determined by the initial conditions. Observing Eq. (8), (11), (12) and (15) one notes that for

$$M_q < 0; \qquad c < 0 \tag{16}$$

$$(\dot{K}_j/K_j) < 0 \tag{17}$$

for all t.

A similar solution can be found for $p_0 \neq 0$.

Numerical Calculations

To check these results, an example that causes Eq. (5) to become positive at $t \approx 0$ was run on a 6-D Computer Program,[4] at the University of Notre Dame. The results are shown in Fig. 1. The data for the computation are presented in Table 1.

Table 1 Data for computations

$C_{M_{p\alpha}}$	$=0$
C_{ℓ_p}	$=-1.76$ 1/rad
C_{M_q}	$=-8.80$ 1/rad
C_{M_α}	$=-0.88$ 1/rad
ω_0	$=10$ rad/sec
V	$=1000$ ft/sec
ρ	$=0.002309$ slugs/ft^3
α_0	$=1.15$ degrees
I_x	$=0.1$ slugs-ft^2
I	$=1.0$ slugs-ft^2
d	$=0.5$ ft
Δt	= integration = 0.001 sec

Conclusions

The final conclusion is that L_p cannot cause λ_j to become positive during flight for any interval of t, and should not, therefore, restrict P_∞ or P_0. However L_p should be considered in the process of evaluating other aerodynamic coefficients from flight data. For this purpose one has to verify that the approximate solution of Eq. (5) is in accordance with the exact solution.

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Bounds for the Critical Load of Certain Elastic Systems under Follower Forces

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Introduction

ELASTIC systems subjected to follower forces are, in general, nonconservative in inature, and they can have both static and dynamic instabilities, called divergence (buckling) and flutter, respectively.[1] However, there is a class of follower force systems, called the divergence type systems, which can have only divergence instability (see, for example Refs. 2-41). In this Note, an approximate Rayleigh-quotient-type solution for a class of divergence-type systems is presented, and the approximate solution is shown to be either an upper or lower bound to the exact solution, depending on the choice of the approximating deflection function.

Elastic System

Consider an undamped, one-dimensional, linearly elastic system occupying a length ℓ. Let the equation of motion be

$$m(\mathrm{d}^2w/\mathrm{d}t^2) + K(w) + P_cF_c(w) + P_nF_n(w) = 0 \tag{1}$$

and the boundary conditions be

$$B(w) = 0 \tag{2}$$

where

x = spatial coordinate of the system
t = time
w = predominant deflection of the system from the equilibrium position, $w = w(x,t)$
m = mass density of the system, $m = m(x)$
P_c = conservative component of the external forces
P_n = nonconservative component of the external forces

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